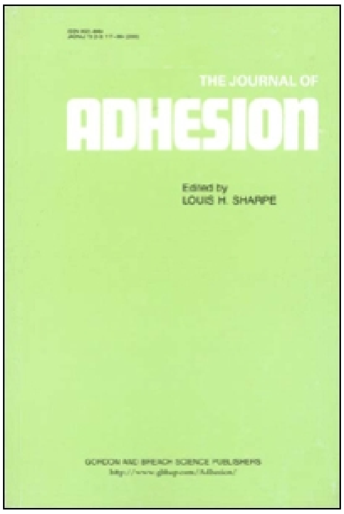

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Stress Distribution and Strength of T-type Adhesive Joint with Reinforcement for Bending Moment†

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The stress distribution in the adhesive layer of T-type adhesive-bonded butt joint between rigid adherends has been measured experimentally, and the equation relating the maximum stress in the adhesive layer to the bending moment applied to the joint and to the joint dimensions was derived. The equation is used to calculate the adhesive strength of a T-type joint from the measured breaking load. These strengths show reasonable agreement with experimental values.

The distribution in the adhesive layer of a T-type adhesive joint with the reinforcement having the section of a right-angled isosceles triangle has been measured experimentally. The strength efficiency of the reinforcement η and the strengthening magnification of the reinforcement μ are discussed geometrically comparing with the equation. The values of η and μ measured by the experiments showed good agreement with the values obtained geometrically.

KEY WORDS Bending moment; reinforced adhesive joints; rigid adherends; strength; stress distribution; T-type adhesive joints.

1 INTRODUCTION

In the first part of this paper, the equation relating the maximum stress in the adhesive layer to the applied bending moment Pl in

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Figure 1 is derived for a T-type adhesive-bonded butt joint between rigid adherends such as steel through the experimental result of the stress distribution of this adhesive layer, and compared with the results of experimental measurement. In the second part, the distributions of stress and reacting moment in a T-type adhesive joint with the reinforcement having the section of the right-angled isosceles triangle shown in Figure 6 are discussed goemetrically. The strengths of both adhesive joints with and without reinforcement are also discussed geometrically and compared with the experimental result.

2 DERIVATION OF EQUATION RELATING MAXIMUM STRESS TO BENDING MOMENT IN T-TYPE ADHESIVE JOINT

Figure 1 shows a T-type adhesive bonded butt joint subjected to an applied bending moment. The adherends A and B are adhesively bonded at the surface JK, and the bending moment Pl is the product of the bending force P and the distance l that P is applied from the adhesive surface. If, as shown in Figure 1, equal and opposite forces P' and P_1 are assumed to act the mid-point O, then the couple Pl comprises P and P_1, and P' becomes the shear force in the adhesive layer.

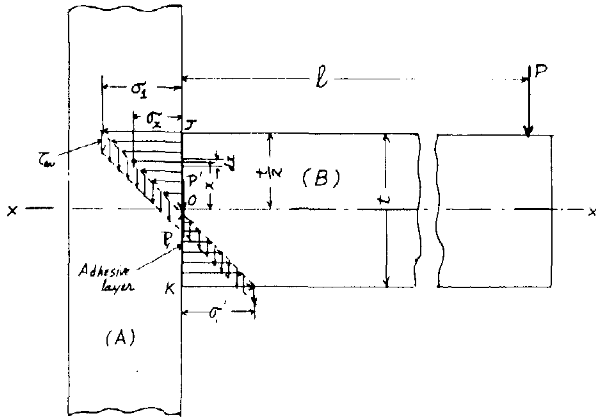

FIGURE 1 T-type adhesive joint with applied bending moment Pl and its schematic stress distribution.

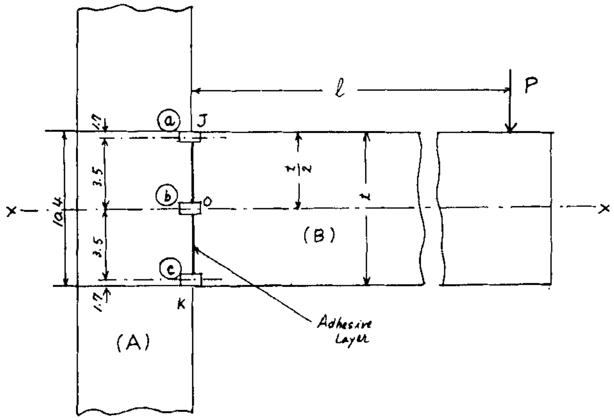

FIGURE 2 Positions of strain gauges to measure strain under applied bending moment for T-type adhesive joint.

Before the maximum stress/bending moment relationship can be derived, it is necessary to know the stress distribution induced in the adhesive as the result of the bending moment. This was determined by placing three strain gauges across the adhesive layer of the T-type joint, as indicated by ⓐ, ⓑ and ⓒ in Figure 2. The strains at these points under different bending forces P applied at $l = 80\,\mathrm{mm}$ are shown in Figure 3. It is clear from this figure that a tensile strain is induced at ⓐ and compression strain at ⓒ, with almost zero strain at ⓑ. As the stress is directly proportional to the strain, the stress σ_x at the distance χ from the mid-point is given by;

$$\sigma_x = \sigma_1 \frac{x^n}{t/2} \tag{1}$$

where σ_1 is the stress at ⓐ or the discance $t/2$ from the mid-point O, t is the vertical length of the adhesive surface and n is almost equal to one. If $n = 1$ in Eq. (1), then the tensile stress distribution across OJ and the compressive stress distribution across OK is shown by the arrows in Figure 1, and σ_1 is the maximum tensile stress. If the area dA at a distance x from O is the product of the breadth b and a small vertical length of adhesive dt, then the

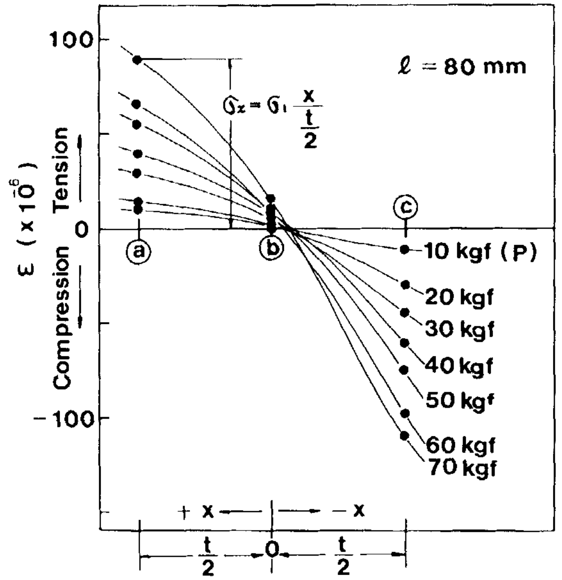

FIGURE 3 Strains detected at ⓐ, ⓑ and ⓒ in T-type adhesive joint under each applied bending force P.

bending moment dM reacting against Pl at dA is given by;

$$dM = \sigma_x \, \mathrm{d}Ax \qquad (2)$$

From Eq. (1)

$$\begin{aligned} dM &= \sigma_1 \frac{x}{t/2} b dx x \\ &= 2b \frac{\sigma_1}{t} x^{(n+1)} dx \end{aligned}$$

The integral value of dM from O to J must be half of the bending moment Pl. Hence;

$$\begin{aligned} \frac{Pl}{2} &= \int_{x=0}^{x=t/2} dM \\ &= \frac{2b\sigma_1}{t} \int_{x=0}^{x=t/2} x^{(n+1)} dx \end{aligned}$$

$$= \frac{2b\sigma_1}{t}\frac{1}{n+2}(x^{n+1})_{x=0}^{x=t/2}$$

$$= \frac{2b\sigma_1}{(n+2)t}\left(\frac{t}{2}\right)^{n+2}$$

$$\therefore \sigma_1 = \frac{2^n(n+2)Pl}{bt^{(n+1)}} \tag{3}$$

When $n = 1$

$$\sigma_1 = \frac{6Pl}{bt^2} \tag{3a}$$

As shown in Figure 1, the average shear stress τ_{av} or the maximum shear stress $\tau_{\max}$ induced at the adhesive surface *JK* by the shearing force P' is;

$$\tau_{\text{av}} = \frac{P'}{bt} = \frac{P}{bt} \tag{4}$$

$$\tau_{\max} = \alpha\tau_{\text{av}} = \frac{P}{bt}\alpha \tag{5}$$

where α is the shear stress concentration factor[1,2] the maximum combined stress or the equivalent tensile stress σ_{e1} (comprising σ_1 and $\tau_{\max}$) is derived from the equation $\sigma_e = (\sigma^2 + \tau^2)^{1/2}$

$$\sigma_{e1} = \left[\left(\frac{2^n(n+2)Pl}{bt^{(n+1)}}\right)^2 + \left(\frac{P}{bt}\alpha\right)^2\right]^{1/2}$$

$$= \frac{P}{bt}\left[\left(\frac{2^n(n+2)}{t^n}\right)^2 + \alpha^2\right]^{1/2} \tag{6}$$

when $n = 1$

$$\sigma_{e1} = \frac{P}{bt}\left[\left(\frac{6l}{t}\right)^2 + \alpha^2\right]^{1/2} \tag{6a}$$

when l/t is greater than 2, $\sigma_1 \gg \tau_{\max}$. Thus $\tau_{\max}$ becomes negligible compared with σ_1, so that Eq. (3) or (3a) is applicable.

3 EXPERIMENTAL VERIFICATION OF THE EQUATION

T-type adhesive-bonded butt joints were prepared between stainless steel adherends (*A* and *B*) with an epoxy-polyamide adhesive at *JK*.

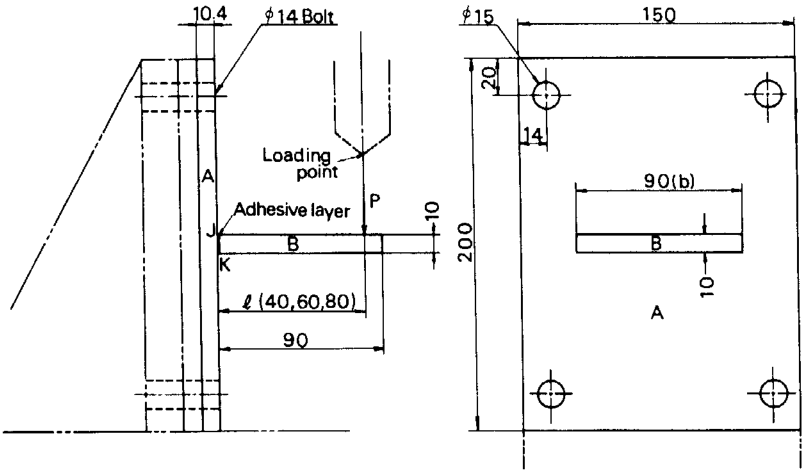

FIGURE 4 Experimental apparatus for breaking the adhesive joint by bending moment Pl.

The joint were mounted in the apparatus shown in Figure 4, and the breaking load P_{max} was measured by increasing the load at distance l of 40, 60 or 80 mm from the adhesive surface. The average value of P_{max} obtained are presented in Table I.

The tensile adhesive strength σ_{j1} of the joint was calculated by substituting the value of P_{max} into Eq. (3a) or (6a) that is,

TABLE I

Bending (or tensile) adhesive strength σ_{j1} for each bending moment $P_{max}l$. For T-type joint

Distance l (mm)	No.	Breaking load P_{max} (kgf)	Mean breaking load $P_{max \cdot av}$ (kgf)	Bending adhesive strength σ_{j1} (kgf/mm^2)
40	1	196	195.5	5.2
	2	195		
	3	—		
60	1	172	169.7	6.8
	2	167.5		
	3	—		
80	1	102	104	5.5
	2	106		
	3	104		

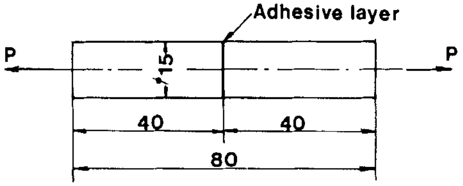

FIGURE 5 Test specimen to measure the tensile adhesive strength.

$\sigma_{j1} = 6P_{\max}l/bt^2$. These values of σ_{j1} are shown in the right-hand column of Table I: they range from 5.2 to 6.8 kgf/mm^2, with the average value being 5.8 kgf/mm^2. Since in these tests the value of l/t is 4 to 8, then $\sigma_1 \gg \tau_1$ and $\sigma_1 = \sigma_{e1}$.

The tensile adhesive strength σ_{jt} obtained using the specimen shown in Figure 5 was 5.2 kgf/mm^2. The reasonable agreement obtained between the theoretical adhesive strength σ_{j1} under bending moment Pl and the tensile experimental value σ_{jt} verifies the use of Eq. (3a) and (6a) to estimate the strength of T-type adhesive bonded butt joint under an applied bending load moment.

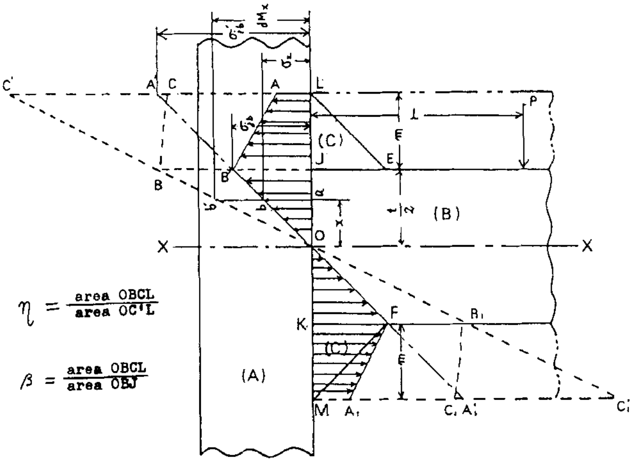

FIGURE 6 T-type adhesive joint with the reinforcement *C* having the section of a right-angled isosceles triangle and its schematic distributions of stress and reacting bending moment under applied bending moment *Pl*.

4 T-TYPE ADHESIVE JOINT WITH REINFORCEMENT

a) Distributions of stress and reacting moment in adhesive layer

To know the stress or strain distribution at the adhesive layers between steel adherends (*A*) and (*B*), (*C*), eleven strain gauges ① ~ ⑪ were placed across the adhesive layer *LM* in Figure 7(a), and also six strain gauges ⑫ ~ ⑰ were placed across the adhesive layers *JE* and *KF* between the steel adherend (*B*) and the reinforcement (*C*). Strains detected by the strain gauges at the adhesive layer *LM* are shown as Figure 8. It is clear from this figure that the stress distribution at the adhesive layer *JK* is similar to that of T-type adhesive joint shown by Figure 1, and that the strain amounts at the layers *LJ* and *KM* between adherend (*A*) and

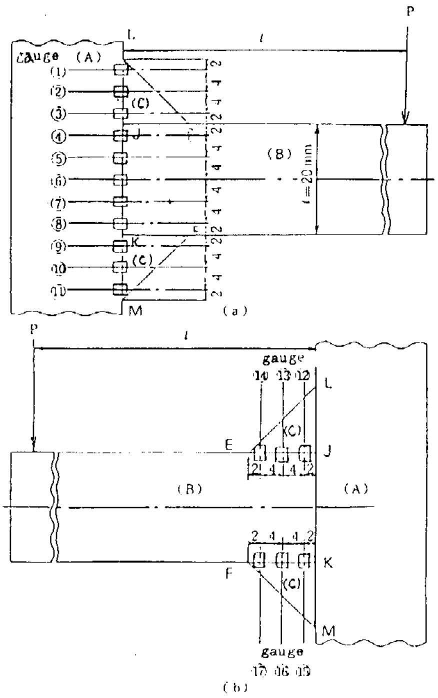

FIGURE 7 Positions of strain gauges attached across the adhesive layer of T-type adhesive joint with the reinforcement *C*.

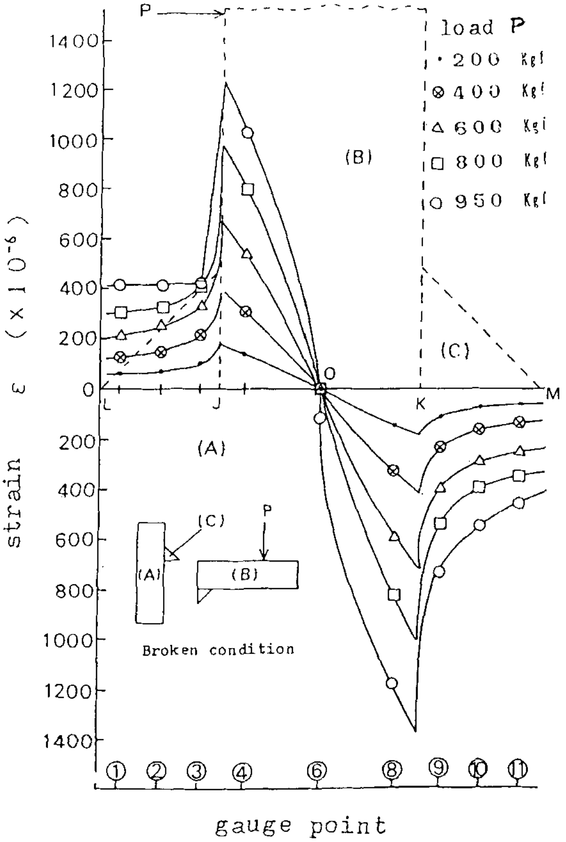

FIGURE 8 Strains detected by strain gauges ① ~ ⑪ of T-type adhesive joint with the reinforcement under each applied bending load *P*.

reinforcement (*C*) are smaller than that at *JK*. The stress distributions at the adhesive layers *JE* and *KF* under bending load are also shown by Figure 9, where σ_t is tensile stress and σ_c is compressive stress. The reason for those phenomenons is assumed to be that there are some relaxations of bonding between adherends (*B*) and (*C*) due to the much lower elastic modulus of the adhesive layers.

We would consider now the distribution of reacting bending moment at adhesive layer *LM* to be balanced to the bending moment *Pl* for a T-type adhesive joint with or without reinforce-

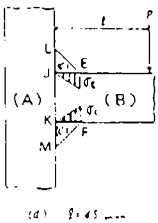

FIGURE 9 Strain distribution at the adhesive layer of *JE* and *KF*.

ment. In the upper half side of the adhesive joint in Figure 6, the reacting bending moment dM_x at minute adhesive area dA having length of dx and width of $b = dx \cdot b$ and x far from mid-point O to be balanced to applied bending moment $Pl/2$ is given by:

$$dM_x = \sigma_x dxbx \tag{7}$$

where σ_x is the tensile stress at the points x far from O. Along the adhesive layer JO, the following relation is obtained;

$$\begin{aligned}\frac{Pl}{2} &= \int_{x=0}^{x=t/2} dM_x \\ &= \int_{x=0}^{x=t/2} \sigma_x \cdot b \cdot x \cdot dx \end{aligned} \tag{8}$$

The value of σ_x is shown geometrically by the distance from OJ to the line OB in Figure 6, and the maximum amount of σ_x is given by BJ. As dM_x is the product of σ_x, dA and x, the value of dM_x is shown geometrically by the length parallel to axis xx of triangle OBJ, and the maximum value is presented by BJ. And then the area of triangle OBJ is equivalent to $\int_{x=0}^{x=t/2} dM_x$. When the thickness of a adherend (B) becomes $(t+2m)$ and the length of adhesive layer is LM. The value of σ_x is shown by the length parallel to the axis xx of triangle OAL, and the maximum value is presented goemetrically by $A'L$. Accrodingly, the value of dM_x is shown geometrically by the length parallel to the axis xx of triangle $OC'L$ and then $\int_{x=0}^{x=t/2} dM_x$ is shown by the area of triangle $OC'L$.

The value of σ_x along the adhesive layer OL of the joint with reinforcement (C) is shown by the length parallel to the axis xx of quadrilateral $OB'AL$ which is similar to that in Figure 8. As there are some relaxations at adhesive layer and some deformation of the reinforcement (*C*), it being an elastic the solid, the amount of dM_x of this case is shown geometrically by the length parallel to the axis *xx* of the quadrilateral *OBCL*, and $\int_{x=0}^{x=1/2+m} dM_x$ is shown by the area of the quadrilateral *OBCL*. The larger the area presented by the integral of dM_x the larger the adhesive strength against the applied bending moment. If $P_{\max}l$ is the applied bending moment to break the T-type adhesive joint with thickness of $(t+2m)$ of adherend (*B*) and $P'_{\max}l$ is that to break T-type adhesive joint with the reinforcement (*C*) with adhesive length of *m*, then their ratio, designated the strength efficiency of reinforcement, η, is given by:

$$\eta = \frac{P'_{\max}l}{P_{\max}l} = \frac{\text{area } OBCL}{\text{area } OC'L} \tag{9}$$

When σ'_{jb} is the stress at *L* of T-type adhesive joint with adhesive length of $(t+2m)$, the area of triangle $OC'L$ is given by:

$$\frac{P_{\max}l}{2} = \frac{\sigma'_{jb}b \cdot (t+2m)^2}{12} \tag{10}$$

When $P_{0\max}l$ is the maximum bending moment for T-type adhesive joint with adhesive layer of *JK* and thickness of adherend (*B*) of *t* and $P'_{\max}l$ is the maximum bending moment for T-type adhesive joint with the reinforcement (*C*) with adhesive length of *m*, their ratio μ is designated the strengthening magnification of reinforcement and is given by:

$$\mu = \frac{P'_{\max}l}{P_{0\max}l} = \frac{\text{area } OBCL}{\text{area } OBJ} \tag{11}$$

b) Experimental

When a bending moment *Pl* was applied to a T-type adhesive joint with 20 or 44 mm thickness and 40 mm width of adherend (*B*), the breaking loads $P_{0\max}(t = 20\text{ mm})$ and $P_{\max}(t = 44\text{ mm})$ for each arm length *l* of 110, 70 and 45 mm were obtained experimentally as shown in Table II. In these cases, the adherends were stainless steel and the adhesive was an epoxy-polyamide.

The tensile or bending adhesive strength σ_{jb} obtained from equation (3a) or (6a) for those T-type adhesive joints were 5.0 to

TABLE II
Experimental results

Type of adhesive bonded joint		Arm length l(mm)	Breaking load $P_{0\max}$ or $P'_{\max}$ (kgf)	Strength efficiency of reinforcement η (%)	Strengthening magnification μ	Adhesive strength σ_{jb}
Without reinforcement T type $t = 20$ mm		110	$P_{0\max}$ 129	100	1.0	5.0
		70	$P_{0\max}$ 238	100	1.0	6.3
		45	$P_{0\max}$ 323	100	1.0	5.5
With reinforcement	m 4 mm	110	$P'_{\max}$ 217	86	1.68	4.6
		70	$P'_{\max}$ 374	80	1.57	5.1
		45	$P'_{\max}$ 459	77	1.42	4.0
	m 8 mm	110	$P'_{\max}$ 372	90	2.88	4.8
		70	$P'_{\max}$ 571	74	2.39	4.7
		45	$P'_{\max}$ 776	76	2.40	4.2
	m 12 mm	110	$P'_{\max}$ 451	73	3.49	3.9
		70	$P'_{\max}$ 754	65	3.16	4.2
		45	$P'_{\max}$ 1,084	72	3.34	4.0
Without reinforcement T type $t = 44$ mm		70	$P_{\max}$ 1,061	100		5.6

$P_{0\max}$ without reinforcement, $P'_{\max}$ with reinforcement

6.3 kgf/mm^2 as shown in Table II. The tensile adhesive strength σ_{jt}, obtained by the test pieces shown in Figure 5, was 4.8 kgf/mm^2 and their bending adhesive strengths, σ_{jf}, were 5.3 to 6.3 kgf/mm^2. The values of σ_{jb}, σ_{jt} and σ_{jf} show good agreement.

For T-type epoxy-polamide adhesive joints between steel adherends (A) and (B) with breadth 40 mm and thickness 20 mm attached adhesively the steel reinforcement (C) with equilateral length m of 4,8 or 12 mm, the breaking bending load $P'_{\max}$ for each arm length l of 110, 70 or 45 mm were obtained as shown in Table II, using the apparatus presenting in Figure 4. The strength efficiencies of reinforcement η were obtained from those values as follows:

$$\eta = \frac{P'_{\max} l}{P_{\max} l} = \frac{P'_{\max}}{P_{\max}} \text{(for same } l \text{ and } t\text{)} \tag{12}$$

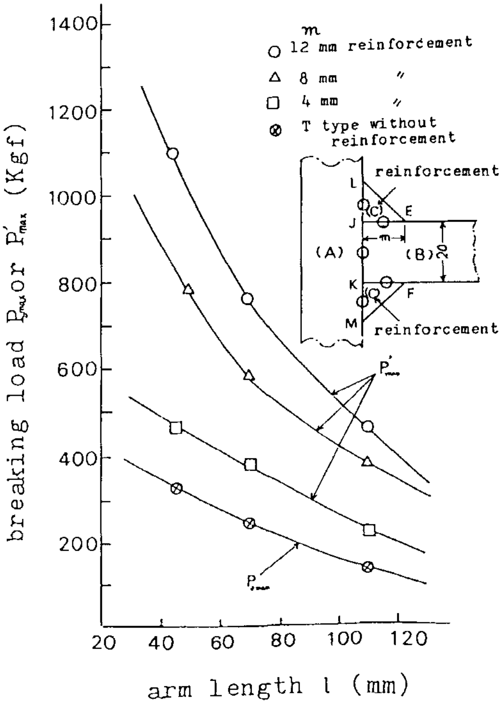

FIGURE 10 Relation between breaking load P'_{max} or P_{0max} and arm length l of bending moment for T-type adhesive joint with each reinforcement C.

The values of η are shown in Table II. Figure 10 shows the relations between the breaking load P_{0max} for 20 mm of t without reinforcement or P'_{max} for each equilateral length m of reinforcement and arm length l of bending moment. Figure 11 shows the relations between adhesive strength of each adhesive joint σ_{jb} obtained from Eq. (3a) assuming that t is $(20+2m)$ or strength efficiency of reinforcement η and arm length l. It seems from this figure that the value of η increases slightly with increase in l.

The strengthening magnifications of reinforcement μ were obtained by the following:

$$\mu = \frac{P'_{max} l}{P_{0max} l} = \frac{P'_{max}}{P_{0max}} \tag{13}$$

where P_{0max} is the breaking bending load of T-type adhesive joint

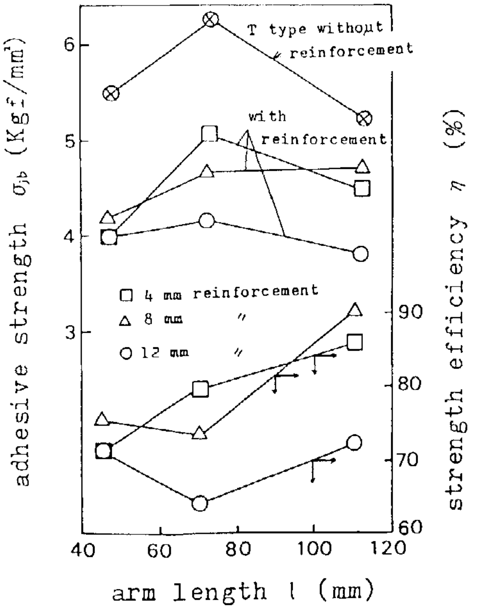

FIGURE 11 Relation between adhesive strength σ_{jb} or strength efficiency of reinforcement η and arm length l of bending moment.

with 20 mm thickness of adherend (B) and $P'_{\max}$ is that with 20 mm thickness of (B) and the reinforcement of 4, 8 and 12 mm of equilateral length m. The values of μ for various cases are shown in Table II. It is clear from the Table that the values of μ are 1.5 for 4 mm of m, 2.55 for 5 mm of m and 3.26 for 12 mm of m. Those values of η or μ are nearly equal to the values obtained from the area ratio given by Eq. (9) or (11) and Figure 6. It seems that there is good agreement between the value of η or μ obtained experimentally and that obtained from the area ratio geometrically.

5 CONCLUSIONS

When a bending force P is applied parallel to and at a distance l from the adhesive surface of a T-type adhesive-bonded butt joint between two steel adherends of thickness t and width b, the maximum tensile stress σ_1, and the maximum equivalent tensile stress σ_{e1} induced at the adhesive layer are given by the equation;

$$\sigma_1 = \frac{2^n(n+2)Pl}{bt^{(n+1)}}$$

$$\sigma_{e1} = \frac{P}{bt}\left[\left(\frac{2^n(n+2)P}{t^2}\right)^2 + \alpha^2\right]^{1/2}$$

where the value of n is 1 when t is less than 10 mm, but becomes greater than 1 when t is greater than 10 mm; α is the shear stress concentration factor and is generally greater than 1 and σ_1 is almost equal to σ_{e1} when l/t is greater than 2.

When a similar bending force is applied to a T-type adhesive joint with a reinforcement having the section of a right-angled isosceles triangle of length m of equilateral, the distributions of stress and reacting bending moment could be presented geometrically. The strength efficiency of reinforcement η and the strengthening magnification of reinforcement μ are given by following:

$$\eta = \frac{P'_{\max}}{P_{\max}}$$

$$\mu = \frac{P'_{\max}}{P_{0\max}}$$

where $P'_{\max}$ is the breaking bending load of the joint with reinforcement of equilateral length m and vertical adhesive length of $(t+2m)$, $P_{\max}$ is that of T-type joint in which the thickness of adherend (B) equals to $(t+2m)$ and $P_{0_{\max}}$ is that of T-type joint with thickness t of adherend (B). The values of η obtained experimentally were 65 to 90% and those of μ were from 1.5 to 3.2, and those values showed reasonable agreement with the values obtained geometrically as the area ratio.

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